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Book Review

Wave Turbulence, by Sergey Nazarenko, Lecture Notes in Physics, Vol. 825
Springer, 2011; ISBN: 978-3-642-15941-1, USD 59.95

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Wave (or Weak) Turbulence (WT) is the study of dispersive waves which are involved in random weakly nonlinear interactions over a broad range of scales in various physical systems. WT is a very spectacular natural phenomenon with applications, e.g., in surface gravity waves, capillary waves, inertial waves, Alfvén waves, nonlinear optics, semiconductors, cluster aggregation, superfluid helium and processes of Bose–Einstein condensation.

The author, Prof. S. Nazarenko, is a theoretical physicist and a widely known specialist of various aspects of turbulence theory and waves. Given the author's reputation, it is not surprising that the book is of high quality.

Wave Turbulence is divided into four parts, beginning with an introductory chapter 1, which briefly describes basic philosophy of the book. Part I, Primer on Wave Turbulence, comprises three chapters. Chapter 2 describes briefly the phenomenology of turbulent flows and places WT in the turbulence theory context. Roughly speaking, turbulence is a highly excited state of a system with infinitely many degrees of freedom that need to be described in a probabilistic sense, whereas, if the interactions between the degrees of freedom are weak, a particular approach called WT can be deployed.

In chapter 3 the key results in the WT theory are shortly presented, using dimensional analysis. First, the notion of the N-wave interaction coefficient is clarified. After introducing this concept, the Kolmogorov–Zakharov spectra and the wave action cascade spectra are derived. Subsequently, such spectra are found for a large variety of physical systems, from capillary waves to waves on elastic plates. Some facts about strong WT are presented and also explained, in terms of critical balance ideas (critical balance refers to a scale-by-scale balance between the linear propagation and nonlinear interaction time scales in anisotropic wave systems). Chapter 4 presents solutions of exercises of some important problems focusing on centroids and wave resonances in 1D and 2D systems.

The applicability of statistical concepts and methods in the study of WT, forms the focus of the Part II, Wave Turbulence Closures. It consists of three chapters. Chapter 5 motivates the subject with brief general overview of nonlinear Schrödinger equation (NLS), which describes waves in nonlinear optics and Bose–Einstein condensates. The time evolution equation for wave function in the discrete Fourier space is derived. Based on the wave function representation in terms of its amplitude and phase, the joint N-mode Probability Density Functions (PDF) for suitable random variables associated with the intensity and the phase factor of the node are determined. After these questions, the basic philosophy of the Random Phase and Amplitude approach (RPA) is outlined and explained. It should be noted that the field is of RPA type if it possesses the following statistical properties: all amplitudes and their phase factors are independent random variables, and the phase factors are uniformly distributed on the unit circle in the complex plane. The remaining sections in this chapter deal with generating function method, wave spectrum, higher one-mode moments, structure functions and the RPA averaging techniques.

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The core of the book is chapter 6. It provides a thorough overview of WT formalism, including the comprehensive and the reduced schemes. The comprehensive WT scheme presents, in 15 steps, a compact programme for building a mathematical skeleton for rigorous WT theory. The reduced approach concerns the one-mode objects. The starting point in illustration of the basic concepts and techniques of WT, is the Petviashvili equation [or Kadomtsev–Petviashvili (KP) equation]. It describes the propagation of long weakly dispersive waves as ion-acoustic waves in plasmas, Rossby waves, etc. The successive sections cover important subjects as conservation laws for the KP equation, Fourier space description and the interaction representation for the KP equation. Based on the amplitude of KP equations and using the formal expansion in the small nonlinearity parameter ε, an appropriate expression for the generating function is obtained (up to order ε^2). Then, the phase averaging and the amplitude averaging are performed, and, in large-box and weak-nonlinearity limits, the evolution equation for the generating function is obtained. Using the inverse Laplace transform, the continuity equation for PDF is derived. On the basis of this result, the kinetic equation for the wave spectrum in symmetrical form is gained.

It is known that hamiltonian formalism is applicable to a broad class of weakly interacting and weakly dissipative wave systems. The following section is devoted to rewriting the KP equation in hamiltonian form and constructing WT from suitable hamiltonian equations. The next section logically continues, basing on concepts presenting in former sections, with discussions of four-wave and higher-order systems (until one-dimensional six-wave structures). The last two sections deal with the extension of the WT approach to multi-mode statistical objects. The clear presentation together with graphical notations (Feynman diagrams) help the reader understand the essence of the described phenomena. Part II is supplemented with chapter 7 which contains additional material and solutions to the majority of exercises pertaining to, e.g., gaussian fields, multi-point moment, invariants of KP equation, Charney-Hasegawa-Mima (CHM) Model.

Part III, Wave Turbulence Predictions, surveys some important issues connected with solutions of the WT closures. Chapter 8 deals with conservation laws in WT systems, invariants and directions of turbulent mode cascades in various wave systems. Chapter 9 presents the most important solutions in WT called Kolmogorov–Zakharov (KZ) spectra. These spectra are discussed for a large variety of physical situations. Locality and stability of the KZ solutions are analysed also, in some detail. Chapter 10 presents a brief, and informal discussion of a future theory of discrete and mesoscopic wave turbulence, whereas chapter 11 addresses the higher-order statistics, the phenomena of WT intermittency (intermittency corresponds to non-gaussianity of WT), and the WT life cycle, where the weak incoherent and the strong coherent wave components coexist, interact and mutual transform. Validity of RPA in the WT closure is also concisely analysed. Part III concludes with a short chapter 12 containing the solutions to the homework problems.

Part IV, Selected Applications, concentrates on describing three main applications of WT, namely: Drift/Rossby WT, Magnetohydrodynamic (MHD) turbulence and WT in Bose–Einstein condensates. It begins, in chapter 13, with a discussion of non-local character of Drift/Rossby WT. On the basis of an analysis of kinetic equation, the suitable evolution equation is derived and its physical properties and consequences are discussed. Subsequently explored are non-local interactions with small-scale zonal flows (ZF), together with weak ZF and strong ZF. The last section refers to a numerical results of the forced-disspated CHM equation.

In chapter 14, a WT theory for incompressible MHD is outlined. Because the theory for such complicated problem is very arduous, the simpler question is considered, namely, reduced MHD model for weak Alfve′n waves. For readers seeking more details, particularly helpful is section about suggested references.

Chapter 15 focuses on the phenomenon of the Bose–Einstein condensation. The condensate is, in fact, a new state of matter, where quantum–mechanical wave functions of ultracold atoms behave as coherent matter waves in the analogous way as coherent light waves in optical systems. For readers who are less familiar with quantum physics, I recommend an excellent book by L. Pitaevskii and

S. Stringari "Bose–Einstein Condensation" (Clarendon Press, Oxford, 2003). Mathematical formalism used in an analysis comprises Gross-Pitaevskii equation, Nordheim kinetic equation, differential approximation model for WT and one-dimensional NLS for optical turbulence. Topics include Rayleigh-Jeans states, KZ cascading states, wave-particle crossover, the route to condensation through vortex annihilations, inhomogeneous WT in systems bounded by a potential, and condensation in one-dimensional optical system with discussion of a WT life cycle. Useful solutions to exercises of specified problems are included also (e.g., direct and inverse cascades in two-dimensional NLS, Madelung transformation, various KZ spectra etc.). The Part ends with an interesting list of problems which could be suited for research themes in a broad spectrum of issues in astro- and geophysical flows, classical and superfluid turbulence, reacting and multi-phase flows, quantum-related questions, etc.

Nazarenko's book is very interesting, competent and well-written. The typographical aspect of the book is rather of good quality, although some misprints of minor importance can be found (e.g., on p. 269). It should prove useful for graduate students and researchers in geophysical fluid mechanics (ocean-related flows, atmospheric flows, etc.), astrophysics, turbulence, plasma physics and physics of condensed matter.